Linear categories: from module categories to derived categories and cluster categories

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- In algebraic topology, one studies derived category of dg-modules over the singular cochain dg-algebra of a simply connected topological space.
- More recently, one studies cluster categories, a categorification of cluster algebras, which are connected to the representation theory of semi-simple Lie groups.

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- Each non-zero object is direct sum of finitely many indecomposable objects.

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2. Galois covering theory.

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where f_i, g_i are radical morphisms.

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   - τ is called *AR-translation.*

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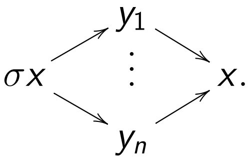
2. This yields a *mesh*



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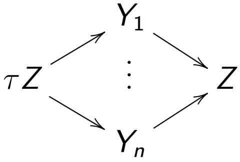

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$$\begin{array}{ccccc} & \nearrow & Y_1 & \searrow & \\ \tau Z & & \vdots & & Z \\ & \searrow & Y_n & \nearrow & \end{array}$$

corresponds to an almost split sequence

$$\tau Z \longrightarrow Y_1 \oplus \cdots \oplus Y_n \longrightarrow Z.$$

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2. Describe the shapes of the components of $\Gamma(\mathcal{A})$.

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$$p : \mathcal{A} \to \mathcal{A}/G : X \mapsto X; f \mapsto f$$

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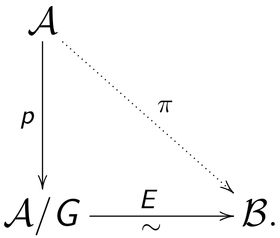

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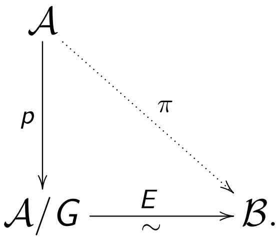

3. Call $\pi : \mathcal{A} \longrightarrow \mathcal{B}$ *Galois G-covering functor*.

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3. If $g \cdot X \notin \Gamma$ for all $X \in \Gamma$ and $(e \neq)\, g \in G$, then

$$\pi(\Gamma) \cong \Gamma.$$

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   - For $M \in \mathrm{ind}\mathcal{T}$, define

$$\mathcal{T}_M := \mathrm{add}\{N \in \mathrm{ind}\mathcal{T} \mid N \not\cong M\}.$$

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Theorem (Koenig, Zhu)

If $\mathcal{T}$ is a cluster-tilting subcategory of $\mathcal{A}$, then

$$\mathrm{mod}\mathcal{T} \cong \mathcal{A}/\mathcal{T}[1].$$

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where f, u minimal left $\mathcal{T}_M$-approximations;

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Objective of Study

Construct more cluster categories of infinite rank.

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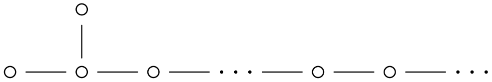

Plan of the rest of this talk

1. $Q = (Q_0, Q_1)$: connected, locally finite, no infinite path.
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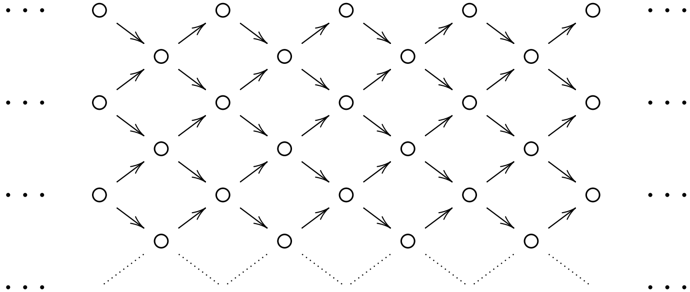

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3. The indecomposable objects of $D^b(\mathrm{rep}(Q))$ are

$$\{M[n] \mid n \in \mathbb{Z}, M \in \mathrm{ind}(\mathrm{rep}(Q))\}.$$

Almost split sequences in $D^b(\mathrm{rep}(Q))$

Theorem (Bautista, Liu, Paquette)

1. Every almost split sequence $X \longrightarrow Y \longrightarrow Z$ in $\mathrm{rep}(Q)$ induces almost split sequences $D^b(\mathrm{rep}(Q))$:

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2. For $a \in Q_0$, $\exists$ almost split sequences in $D^b(\mathrm{rep}(Q))$:

$$I_a[n-1] \longrightarrow (\oplus_{a_i \to a} I_{a_i}[n-1]) \oplus \left(\oplus_{a \to b_j} P_{b_j}[n]\right) \longrightarrow P_a[n],$$

for $n \in \mathbb{Z}$.

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3. AR-translation τ_D is automorphism of $D^b(\mathrm{rep}(Q))$.

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Theorem (Keller, Buan-Iyama-Reiten-Scott)

$\mathscr{C}(Q)$ is triangulated, which is a cluster category in case Q is is finite.

Conjecture

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Remark

It suffices to show that the quiver of any cluster-tilting subcategory in $\mathscr{C}(Q)$ has no oriented cycle of length one or two.

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3. The number of such AR-components is generally finite.